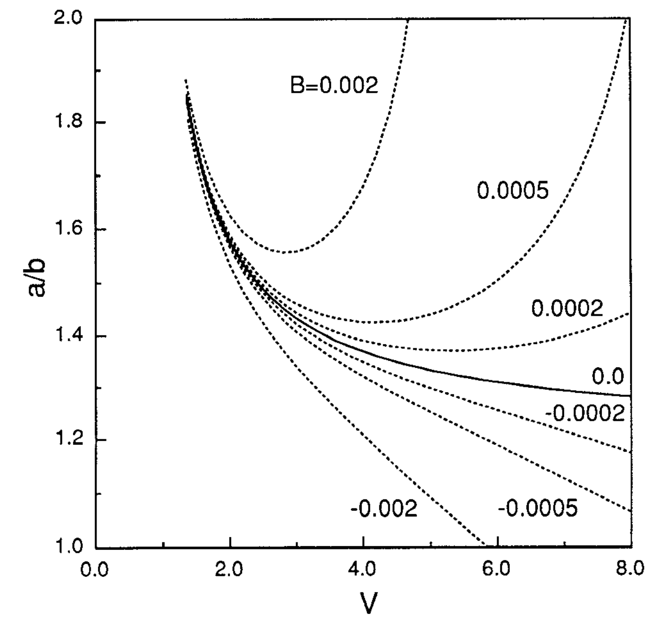

Fig. 3. Aspect ratio required for achieving a specified modal birefringence B in a strip waveguide with $n_1 = 3.44$, $n_2 = 3.40$, and $n_3 = 1.0$. B is the difference between the normalized propagation constants of the E^x_{11} and E^y_{11} modes.

IV. Conclusion

A simple and accurate relation has been derived to describe the dispersion characteristics of a strip waveguide. Apart from providing a much more efficient way for calculating dispersion, this relation brings out explicitly many physical properties of the waveguide and should be useful for the study and design of strip waveguides. A simple application of this relation has revealed an interesting property of the waveguide, namely, that it is always possible to make the two polarized modes of the waveguide degenerate by using an appropriate aspect ratio for the strip.

Acknowledgment

The useful discussion with P. E. Ciddor is gratefully acknowledged.

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Microwave Measurement of the Dielectric Constant of High-Density Polyethylene

Karlheinz Seeger

Abstract—**By applying a new microwave technique which involves observing interference fringes in transmission, metallizing the sample faces adjacent to the waveguide, and thus using the sample as a dielectric-filled metallic waveguide, the real part of the dielectric function of high-density polyethylene has been determined as 2.34 at room temperature and 2.29 at liquid nitrogen temperature (77 K) for the frequency range from 26.5 to 40 GHz.**

I. Introduction

Interest in the dielectric behavior of polymers at microwave frequencies has usually been focused on the loss tangent because of low-loss technical applications [1]. Not until quite recently has a precision method for the microwave measurement of the real part of the complex dielectric function been applied to semi-insulating semiconductors [2], [3]. In the present paper the real part of the dielectric constant, ϵ, of high-density polyethylene measured by this microwave interference technique will be reported. Since the loss tangent of this polymer is only of the order of 10^{-4}, the variation of ϵ over a microwave frequency band is of the same order of magnitude, which means that this variation can be neglected in the evaluation of an interference spectrum, quite similar to the case of high-resistivity semiconductors.

II. Experimental Technique

The experimental technique has been reported in detail in [2]. Therefore it suffices to explain it in principle only.

The sample with plane-parallel front and rear ends completely fills a rectangular waveguide for a length d. A Q-band waveguide for 26.5 to 40 GHz has been found convenient for

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The author is with the Ludwig Boltzmann Institut für Festkörperphysik and the Institut für Festkörperphysik der Universität Wien, Kopernikusgasse 15, A-1060 Vienna, Austria.

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two reasons: the sample dimensions, which enter critically into the evaluation of ϵ, can still be determined with high precision; and the microwave sweep oscillator (Hewlett Packard 8690 B) and the waveguide components are standard equipment in microwave laboratories. The sample thickness, d, should be several times the average wavelength in the sample material such that when the frequency varies through the passband of the waveguide, the transmitted power shows between three and five interference maxima. It is mainly from the positions of these maxima that ϵ is determined. Sample thicknesses d of about 2, 2.5, and 3 cm have been used. The incident power can be taken as the sum of the transmitted and reflected powers, since absorption in the sample and in the waveguide was negligibly small. For the evaluation of ϵ, the transmission, T, is evaluated from

$$T = \frac{P_{\text{trans}}}{P_{\text{trans}} + P_{\text{refl}}} = \frac{1}{1 + C \cdot \sin^2 \theta} \tag{1}$$

where P_{trans} and P_{refl} are the transmitted and reflected power levels and

$$C = \frac{(\epsilon - 1)^2}{4(\epsilon - p)(1 - p)} \tag{2}$$

$$p = (\lambda_0 / 2a)^2 \tag{3}$$

and

$$\theta = \frac{2\pi d}{\lambda_0}\sqrt{\epsilon - p}\,. \tag{4}$$

The term p corrects for the fact that the sample is mounted inside a waveguide having an internal dimension, a, nearly equal to the width of the broad sample face. The microwave frequency, ν enters by means of the free-space wavelength, $\lambda_0 = c/\nu$, with $c \approx 3 \times 10^8$ m/s.

Lynch [1] considered effects of overmoding and of air gaps between the sample and the waveguide. Although this effect was considered in [2] as a cause for side peaks in the interference spectrum, it was later found that these side peaks arise from reflections at the waveguide flanges between the sample and thermistor used for measuring the transmitted power, P_{trans}. The effect of unwanted reflections has meanwhile been reduced by mounting a unidirectional transmission line in front of this thermistor. The effect of air gaps has been eliminated by evaporating first chromium and then gold on the side faces of the sample. The metallized faces act as a slightly smaller waveguide; therefore the corresponding sample dimension must be substituted for a in (3).

The polyethylene samples were prepared from a commercially available material (Lupolen 1810A) which was pressed into plane-parallel plates at 150°C (by courtesy of Dr. Hellmann, Deutsches Kunststoff-Institut, Darmstadt). Rectangular bars with the cross section required to fill the waveguide and with different thicknesses were cut to size on a milling machine. The sample thicknesses, d, are given in Fig. 1 The width, a, of the 24.36-mm-thick sample was 6.76 mm, and the widths of two other samples were 6.79 mm.

III. Results

Fig. 1 shows interference data points taken every 0.25 GHz. Obtaining these data involved reading the Hewlett Packard R532A frequency meter each time. The determination of ϵ from these data relies on a trial-and-error procedure: curves have to be plotted from (1) for various values of ϵ until the best fit is found. The spectra for three samples of compacted polyethylene of thicknesses 19.32 mm, 24.36, and 29.44 mm are shifted vertically in the figure. The theoretical curves are based on $\epsilon = 2.34$, 2.33, and 2.34 in the order given above. The average value of ϵ is 2.34 at room temperature. This is to be compared with a value of 2.34 observed at 1 MHz and 20°C with high-density polyethylene [4]. The comparison shows that in the frequency range from 1 MHz to 40 GHz, the dielectric constant of polyethylene shows no dispersion within experimental error limits. This would indicate that there is practically no absorption in this frequency range, as can be seen from the well-known Kramers–Kronig relations because for poly (2-vinyl pyridine) the $\epsilon(\omega)$ spectrum has been shown to be flat from 10^4 to 10^{12} Hz with the imaginary part ϵ'' of ϵ being zero in this range [5].

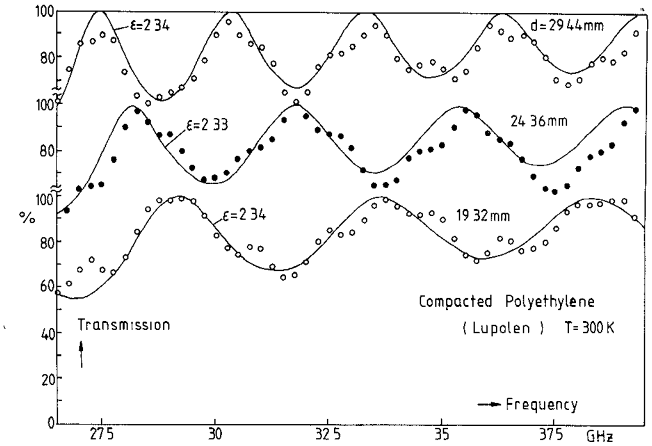

Fig. 1. Measured interference spectra of compacted polyethylene at 300 K for three different sample thicknesses, d, and theoretical curves for dielectric constants, ϵ, with values indicated adjacent to each curve. Notice the vertical shift in transmission scale for different samples.

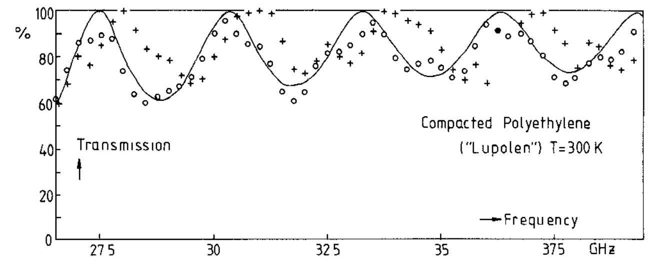

Fig. 2. Uppermost interference spectrum of Fig. 1 (open circles and theoretical curve) compared with data (crosses) obtained before evaporating chromium/gold layers on the sample faces which come into contact with the waveguide walls.

Very similar results have been obtained in an arrangement where the sample was cooled with liquid nitrogen (77 K). Of course, the reduced dimensions of the samples produced by cooling were taken into account. The result for ϵ is 2.29, which is 2% smaller than the room-temperature value. This result corresponds approximately to a reduction of 1.7% in the linear sample dimension upon cooling to 77 K.

Fig. 2 shows again the uppermost curve of Fig. 1 together with the data points represented by open circles. As mentioned above, these data were taken using the thickest of the three samples after it had been covered by a thin layer of Cr/Au on the surfaces which come into contact with the waveguide walls. For a comparison, the crosses represent data taken with the same sample but before the evaporation with Cr/Au was done. The large difference between the two sets of data is believed to be due to the effect of a series capacitance produced by the imperfect fit of the sample in the waveguide [2].

Experiments with high-density polyethylene obtained from a different manufacturer (Dehoplast) yielded the same results.

IV. Conclusions

The method of using interference spectra for determining the dielectric constant, which previously has been developed for semiconductors with dielectric constants of 10–20, has been shown to also yield reliable data for a plastic material, polyethylene, with a dielectric constant of only about 2. Since, for most polymers, the dispersion is very small in the radio-frequency to microwave range and the microwave measurement is both very simple and accurate, the interference method may find technical application in material characterization. Prerequisites are, of course, a low loss tangent and, hence, good transmission through the thick sample.

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The chromium–gold evaporation of the samples by Dr. V. Schlosser is gratefully acknowledged. The author is indebted to Dr. W. Meyer, Max-Planck-Institut für Polymerforschung, Mainz, and Dr. Köhler, BASF, Ludwigshafen, for supplying the Lupolen 1810 A material and to Dr. Hellmann, Deutsches Kunststoff-Institut, Darmstadt, for pressing plane-parallel plates from Lupolen 1810 A. The author wishes to express his deep gratitude to Prof. Dr. G. Harbeke, Paul Scherrer Institute, Zürich, for donating the samples and for a revision of the manuscript.

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Equivalence of Propagation Characteristics for the Transmission-Line Matrix and Finite-Difference Time-Domain Methods in Two Dimensions

N. R. S. Simons and E. Bridges

Abstract —**In previous papers an equivalence between the TLM and FD-TD methods has been established by altering the definitions of field components and operation of the TLM algorithm such that the appropriate finite-difference expressions are satisfied. In this paper the equivalence of propagation characteristics for the TLM and FD-TD methods in two dimensions is discussed. Propagation analysis of a TLM shunt node complete with permittivity and loss stubs, and dispersion analysis of the two-dimensional FD-TD method in an arbitrary medium are performed and yield dispersion relations. The relations are identical when the FD-TD method is operated at the upper limit of its stability range.**

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The authors are with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2.
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I. Introduction

The finite-difference time-domain (FD–TD) method, devised by Yee [1], is based on central difference approximations of Maxwell's curl equations. The method has been successfully applied to a variety of problems, as summarized by Taflove and Umashankar [2]. The transmission-line matrix (TLM) method, pioneered by Johns and Beurle [3], is based on a physical model of wave propagation and has also been extensively applied to the analysis of electromagnetic field problems [4]. Both are time-domain numerical techniques capable of simulating Maxwell's equations in arbitrary media.

Previous papers by Johns and Butler [5] and Johns [6] have shown that TLM models for diffusion and electromagnetic field problems can be operated in such a way that they are equivalent to finite-difference methods. The implications of operating the TLM models under these circumstances have been discussed by Johns [6], with comments by Gwarek [7]. In [5] and [6], the TLM method is formulated in terms of global scattering and connection matrices. Johns has stated that a great deal of flexibility exists in the operation of a TLM algorithm and the definitions of field components in terms of the pulses traveling along the elemental transmission lines. Johns describes the manner in which a mesh of three-dimensional expanded nodes can be operated such that it satisfies the three-dimensional Yee algorithm [6]. A similar analysis has been performed in which a mesh of unloaded two-dimensional shunt nodes is shown to satisfy the Yee algorithm in two dimensions [8]. The equivalence is obtained by altering the definitions for field quantities in the TLM mesh.

The purpose of this paper is to show that, mathematically, the propagation characteristics of the two methods are identical under certain circumstances, regardless of the definitions of field quantities and operation of the TLM model.

II. Dispersion Relation of FD–TD Method

For finite-difference approximations of the wave equation, the dispersion relation is obtained by substituting the mathematical representation of a plane wave into the difference approximation of the wave equation. A general discussion of dispersion in finite-difference models of the wave equation has been provided by Trefethen [9]. For two-dimensional field distributions (independent of the z direction, and selecting $H_z = 0$) Maxwell's curl equations can be combined to yield the following wave equation for E_z:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \sigma\mu \frac{\partial E_z}{\partial t} + \epsilon\mu \frac{\partial^2 E_z}{\partial t^2} \tag{1}$$

for a medium of permittivity ϵ, permeability μ and conductivity σ. Expression (1) can be discretized using central difference approximations to obtain

$$\begin{aligned} &\frac{E_z^{t_0}(x_0+\Delta l, y_0) - 2E_z^{t_0}(x_0, y_0) + E_z^{t_0}(x_0 - \Delta l, y_0)}{\Delta l^2} \\ &\quad + \frac{E_z^{t_0}(x_0, y_0+\Delta l) - 2E_z^{t_0}(x_0, y_0) + E_z^{t_0}(x_0, y_0 - \Delta l)}{\Delta l^2} \\ &= \sigma\mu \frac{E_z^{t_0+\Delta t}(x_0, y_0) - E_z^{t_0-\Delta t}(x_0, y_0)}{2\Delta t} \\ &\quad + \epsilon\mu \frac{E_z^{t_0+\Delta t}(x_0, y_o) - 2E_z^{t_o}(x_o, y_o) + E_z^{t_o-\Delta t}(x_o, y_o)}{\Delta t^2}. \end{aligned} \tag{2}$$